

Radiative Generation of Leptonic CP Violation

Shu Luo and **Jianwei Mei**

*Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918, Beijing 100049, China*

Zhi-zhong Xing

*CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
and Institute of High Energy Physics, Chinese Academy of Sciences,
P.O. Box 918, Beijing 100049, China **
(Electronic address: xingzz@mail.ihep.ac.cn)

Abstract

Three CP-violating phases of the 3×3 lepton flavor mixing matrix V are entangled with one another in the renormalization-group evolution from the seesaw scale ($\Lambda_{\text{SS}} \sim 10^{14}$ GeV) to the electroweak scale ($\Lambda_{\text{EW}} \sim 10^2$ GeV). Concerning the Dirac phase δ , we show that $\delta = 90^\circ$ at Λ_{EW} can be radiatively generated from $\delta = 0^\circ$ at Λ_{SS} in the minimal supersymmetric standard model, if three neutrino masses are nearly degenerate. As for the Majorana phases ρ and σ , it is also possible to radiatively generate $\rho = 90^\circ$ or $\sigma = 90^\circ$ at Λ_{EW} from $\rho = 0^\circ$ or $\sigma = 0^\circ$ at Λ_{SS} . The one-loop renormalization-group equations for the Jarlskog invariant and two off-diagonal asymmetries of V are derived, and their running behaviors from Λ_{SS} to Λ_{EW} are numerically illustrated.

PACS number(s): 14.60.Pq, 13.10.+q, 25.30.Pt

Typeset using REVTeX

*Mailing address

I. INTRODUCTION

Recent solar [1], atmospheric [2], reactor (KamLAND [3] and CHOOZ [4]) and accelerator (K2K [5]) neutrino oscillation experiments have provided us with very robust evidence that neutrinos are massive and lepton flavors are mixed. The phenomenon of lepton flavor mixing can be described by a 3×3 unitary matrix V , commonly referred to as the Maki-Nakagawa-Sakata (MNS) matrix [6]. A useful parametrization of V reads [7]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$ (for $ij = 12, 23$ and 13). Note that the CP-violating phase δ governs the strength of CP or T violation in normal neutrino oscillations, and it does not appear in the expression of $\langle m \rangle_{ee}$ — the effective mass term of the neutrinoless double-beta decay¹. On the other hand, the CP-violating phases ρ and σ only affect $\langle m \rangle_{ee}$, and they can be rotated away if the massive neutrinos are Dirac particles. Current experimental data indicate $\theta_{12} \approx 33^\circ$, $\theta_{23} \approx 45^\circ$ and $\theta_{13} < 10^\circ$ [8], but the phase parameters δ , ρ and σ are entirely unrestricted. A variety of new neutrino experiments are underway, not only to measure the smallest mixing angle θ_{13} and the Dirac phase δ , but also to constrain the Majorana phases ρ and σ .

While neutrino masses and lepton flavor mixing parameters can be measured at low-energy scales, their origin is most likely to depend on some unspecified interactions at a superhigh energy scale. For instance, the existence of very heavy right-handed neutrinos and lepton number violation may naturally explain the smallness of left-handed neutrino masses via the famous seesaw mechanism [9] at the scale $\Lambda_{\text{SS}} \sim 10^{14}$ GeV. Below this seesaw scale, the effective Lagrangian for lepton Yukawa interactions can be written as

$$-\mathcal{L} = \overline{E}_L H_1 Y_l l_R - \frac{1}{2} \overline{E}_L H_2 \cdot \kappa \cdot H_2^c E_L^c + \text{h.c.} \quad (2)$$

in the minimal supersymmetric standard model (MSSM)², where E_L denotes the leptonic $SU(2)_L$ doublets, H_1 and H_2 are the Higgs fields, l_R denotes the right-handed charged leptons, $H_2^c \equiv i\sigma^2 H_2^*$ and $E_L^c \equiv i\sigma^2 \mathcal{C} \overline{E}_L^T$ with \mathcal{C} being the Dirac charge-conjugate matrix. After spontaneous gauge symmetry breaking at the electroweak scale $\Lambda_{\text{EW}} \sim 10^2$ GeV, we arrive at the charged lepton mass matrix $M_l = v Y_l \cos \beta$ and the effective Majorana neutrino mass matrix $M_\nu = v^2 \kappa \sin^2 \beta$, where $v \approx 174$ GeV and $\tan \beta$ is the ratio of the vacuum expectation values of H_2 and H_1 in the MSSM. The lepton flavor mixing matrix V arises from the mismatch between the diagonalization of Y_l (or M_l) and that of κ (or

¹Namely, $\langle m \rangle_{ee} = |m_1 c_{12}^2 c_{13}^2 e^{2i\rho} + m_2 s_{12}^2 c_{13}^2 e^{2i\sigma} + m_3 s_{13}^2|$ is independent of δ .

²For the sake of simplicity, we assume the supersymmetry breaking scale Λ_{SUSY} to be close to the electroweak scale Λ_{EW} . Even if $\Lambda_{\text{SUSY}}/\Lambda_{\text{EW}} \sim 10$, the relevant RGE running effects between these two scales are negligibly small for the physics under consideration.

M_ν). In the flavor basis where Y_l is real and diagonal, V directly links the neutrino mass eigenstates (ν_1, ν_2, ν_3) to the neutrino flavor eigenstates $(\nu_e, \nu_\mu, \nu_\tau)$. The physical parameters at Λ_{SS} and Λ_{EW} are related by the renormalization group equations (RGEs) [10]. It has been shown that the RGE evolution between these two scales may have significant effects on the mixing angle θ_{12} and the CP-violating phases δ , ρ and σ [11–15], in particular if the masses of three light neutrinos are nearly degenerate. It has also been noticed by Casas *et al* [12] and Antusch *et al* [13] that a CP-violating phase can be radiatively generated due to the RGE running from Λ_{SS} to Λ_{EW} .

The reason for the radiative generation of a CP-violating phase is simply that three phases of V are entangled with one another in the RGEs. In other words, the running behavior of δ depends on a non-linear function of δ , ρ and σ . It is therefore possible to generate a non-zero value of δ at Λ_{EW} even if $\delta = 0$ holds at Λ_{SS} , provided the initial values of ρ and σ are not vanishing. This observation opens a new and interesting window to understand possible connection between the phenomena of CP violation at low- and high-energy scales; e.g., the phase parameter governing the strength of CP violation in a long-baseline neutrino oscillation experiment could be radiatively generated from those CP-violating phases which control the leptogenesis of right-handed neutrinos [16] at the seesaw scale.

The main purpose of this paper is to analyze the radiative generation of three CP-violating phases (δ, ρ, σ) via the one-loop RGE running effects from Λ_{SS} to Λ_{EW} . Our work is different from the one done in Refs. [12] and [13] at least in the following aspects:

- The phase convention of V taken by us in Eq. (1) forbids the phase parameter δ to appear in the effective mass of the neutrinoless double-beta decay $\langle m \rangle_{ee}$. Hence it makes sense to refer to δ as the Dirac phase. In contrast, the so-called “Dirac” phase in the “standard” parametrization of V used by Casas *et al* [12] and Antusch *et al* [13] enters the expression of $\langle m \rangle_{ee}$ and is not purely of the Dirac nature. Our RGEs for three CP-violating phases turn out to be remarkably different from theirs.
- We focus on the possibilities to generate $\delta = 90^\circ$, $\rho = 90^\circ$ or $\sigma = 90^\circ$ at Λ_{EW} from $\delta = 0^\circ$, $\rho = 0^\circ$ or $\sigma = 0^\circ$ at Λ_{SS} . While $\delta = 90^\circ$ might imply “Maximal” CP violation in some sense [17], $\rho = 90^\circ$ or $\sigma = 90^\circ$ will lead to a kind of large cancellation in $\langle m \rangle_{ee}$. The RGE running of δ is of particular interest, because it means the radiative generation of leptonic unitarity triangles in the complex plane; i.e., three overlapped lines (sides) at Λ_{SS} can evolve into a triangle at Λ_{EW} .
- The one-loop RGEs for the Jarlskog parameter (denoted by \mathcal{J}) [18] and two off-diagonal asymmetries of V (denoted as \mathcal{A}_L and \mathcal{A}_R) [19] are derived, and their running behaviors from Λ_{SS} to Λ_{EW} are illustrated. These rephasing-invariant quantities measure the strength of Dirac-type CP violation and the geometrical structure of V , respectively. For example, $\mathcal{A}_L = 0$ (or $\mathcal{A}_R = 0$) would imply V to be symmetric about its V_{e1} - $V_{\mu 2}$ - $V_{\tau 3}$ (or V_{e3} - $V_{\mu 2}$ - $V_{\tau 1}$) axis.

Because the radiative corrections to V are expected to be insignificant in the case that three light neutrinos have a strong mass hierarchy and $\tan\beta$ takes small or mild values, we shall mainly consider the quasi-degenerate neutrino mass spectrum in our numerical calculations.

Section II is devoted to the one-loop RGEs for three neutrino masses, three lepton flavor mixing angles, three CP-violating phases and three rephasing-invariant quantities of V . A

detailed numerical analysis of radiative corrections to the Dirac and Majorana phases is presented in section III, where the RGE evolution of \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R is also analyzed. We give a brief summary of our results with some concluding remarks in Section IV.

II. RGEs FOR PHYSICAL PARAMETERS

Below the seesaw scale, the effective neutrino coupling matrix κ obeys the following one-loop RGE in the MSSM:

$$16\pi^2 \frac{d\kappa}{dt} = \alpha\kappa + (Y_l Y_l^\dagger) \kappa + \kappa (Y_l Y_l^\dagger)^T \quad (3)$$

with $t \equiv \ln(\mu/\Lambda_{\text{SS}})$ and

$$\alpha = -\frac{6}{5}g_1^2 - 6g_2^2 + 6(y_u^2 + y_c^2 + y_t^2), \quad (4)$$

where g_1 and g_2 denote the gauge couplings, and y_f (for $f = u, c, t$) stand for the Yukawa couplings of up-type quarks. In the flavor basis where Y_l is real and diagonal, we have $\kappa = V\bar{\kappa}V^T$ with $\bar{\kappa} = \text{Diag}\{\kappa_1, \kappa_2, \kappa_3\}$. The neutrino masses at Λ_{EW} read as $m_i = v^2 \kappa_i \sin^2 \beta$ (for $i = 1, 2, 3$). It is then possible to derive the RGEs for $(\kappa_1, \kappa_2, \kappa_3)$, $(\theta_{12}, \theta_{23}, \theta_{13})$ and (δ, ρ, σ) from Eq. (3), similar to the work done in Refs. [12–15].

For simplicity, we neglect the small contributions of y_e and y_μ to Eq. (3) in our calculations. The RGEs of κ_i (for $i = 1, 2, 3$) turn out to be

$$\begin{aligned} \frac{d\kappa_1}{dt} &= \frac{\kappa_1}{16\pi^2} \left[\alpha + 2y_\tau^2 (s_{12}^2 s_{23}^2 - 2c_\delta c_{12} c_{23} s_{12} s_{23} s_{13} + c_{12}^2 c_{23}^2 s_{13}^2) \right], \\ \frac{d\kappa_2}{dt} &= \frac{\kappa_2}{16\pi^2} \left[\alpha + 2y_\tau^2 (c_{12}^2 s_{23}^2 + 2c_\delta c_{12} c_{23} s_{12} s_{23} s_{13} + c_{23}^2 s_{12}^2 s_{13}^2) \right], \\ \frac{d\kappa_3}{dt} &= \frac{\kappa_3}{16\pi^2} \left[\alpha + 2y_\tau^2 c_{23}^2 c_{13}^2 \right], \end{aligned} \quad (5)$$

where $c_\delta \equiv \cos \delta$. The RGEs of θ_{ij} (for $ij = 12, 23, 13$) are found to be

$$\begin{aligned} \frac{d\theta_{12}}{dt} &= \frac{y_\tau^2}{16\pi^2} \left\{ \frac{c_{(\rho-\sigma)}}{\zeta_{12}} \left[c_{(\rho-\sigma)} c_{12} s_{12} (s_{23}^2 - c_{23}^2 s_{13}^2) - (c_{(\delta+\rho-\sigma)} c_{12}^2 - c_{(\delta-\rho+\sigma)} s_{12}^2) c_{23} s_{23} s_{13} \right] \right. \\ &\quad + \zeta_{12} s_{(\rho-\sigma)} \left[s_{(\rho-\sigma)} c_{12} s_{12} (s_{23}^2 - c_{23}^2 s_{13}^2) - (s_{(\delta+\rho-\sigma)} c_{12}^2 + s_{(\delta-\rho+\sigma)} s_{12}^2) c_{23} s_{23} s_{13} \right] \\ &\quad - \left[\frac{c_\rho}{\zeta_{13}} (c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13}) - \zeta_{13} s_\rho (s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13}) \right] c_{23} s_{12} s_{13} \\ &\quad \left. - \left[\frac{c_\sigma}{\zeta_{23}} (c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13}) - \zeta_{23} s_\sigma (s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13}) \right] c_{12} c_{23} s_{13} \right\}, \\ \frac{d\theta_{23}}{dt} &= \frac{y_\tau^2 c_{23}}{16\pi^2} \left\{ \left[\frac{c_{(\delta-\rho)}}{\zeta_{13}} (c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13}) + \zeta_{13} s_{(\delta-\rho)} (s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13}) \right] s_{12} \right. \\ &\quad \left. + \left[\frac{c_{(\delta-\sigma)}}{\zeta_{23}} (c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13}) + \zeta_{23} s_{(\delta-\sigma)} (s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13}) \right] c_{12} \right\}, \end{aligned}$$

$$\begin{aligned} \frac{d\theta_{13}}{dt} = & \frac{y_\tau^2 c_{23}}{16\pi^2} \left\{ - \left[\frac{c_\rho}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) - \zeta_{13} s_\rho \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \right] c_{12} c_{13} \right. \\ & \left. + \left[\frac{c_\sigma}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) - \zeta_{23} s_\sigma \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right] c_{13} s_{12} \right\}, \quad (6) \end{aligned}$$

in which $\zeta_{ij} \equiv (\kappa_i - \kappa_j)/(\kappa_i + \kappa_j)$ (for $ij = 12, 23, 13$), $c_A \equiv \cos A$ and $s_A \equiv \sin A$ (for $A = \delta, \rho, \sigma, \delta - \rho, \delta - \sigma, \rho - \sigma, \delta + \rho - \sigma, \delta - \rho + \sigma$). In addition, the RGEs of δ, ρ and σ are obtained as follows:

$$\begin{aligned} \frac{d\delta}{dt} = & \frac{y_\tau^2}{16\pi^2} \left\{ \frac{s_{(\rho-\sigma)}}{\zeta_{12}} \left[c_{(\rho-\sigma)} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(c_{(\delta+\rho-\sigma)} c_{12}^2 - c_{(\delta-\rho+\sigma)} s_{12}^2 \right) \frac{c_{23} s_{23} s_{13}}{c_{12} s_{12}} \right] \right. \\ & - \zeta_{12} c_{(\rho-\sigma)} \left[s_{(\rho-\sigma)} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(s_{(\delta+\rho-\sigma)} c_{12}^2 + s_{(\delta-\rho+\sigma)} s_{12}^2 \right) \frac{c_{23} s_{23} s_{13}}{c_{12} s_{12}} \right] \\ & + \frac{1}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) \left[\frac{s_\rho c_{23}}{c_{12} s_{13}} \left(c_{12}^2 - s_{12}^2 s_{13}^2 \right) - s_{(\delta-\rho)} \frac{s_{12}}{s_{23}} \left(c_{23}^2 - s_{23}^2 \right) \right] \\ & + \zeta_{13} \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \left[\frac{c_\rho c_{23}}{c_{12} s_{13}} \left(c_{12}^2 - s_{12}^2 s_{13}^2 \right) + c_{(\delta-\rho)} \frac{s_{12}}{s_{23}} \left(c_{23}^2 - s_{23}^2 \right) \right] \\ & - \frac{1}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) \left[\frac{s_\sigma c_{23}}{s_{12} s_{13}} \left(s_{12}^2 - c_{12}^2 s_{13}^2 \right) + s_{(\delta-\sigma)} \frac{c_{12}}{s_{23}} \left(c_{23}^2 - s_{23}^2 \right) \right] \\ & \left. - \zeta_{23} \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \left[\frac{c_\sigma c_{23}}{s_{12} s_{13}} \left(s_{12}^2 - c_{12}^2 s_{13}^2 \right) - c_{(\delta-\sigma)} \frac{c_{12}}{s_{23}} \left(c_{23}^2 - s_{23}^2 \right) \right] \right\}, \\ \frac{d\rho}{dt} = & \frac{y_\tau^2}{16\pi^2} \left\{ \frac{s_{(\rho-\sigma)}}{\zeta_{12}} \left[c_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(c_{(\delta+\rho-\sigma)} c_{12}^2 - c_{(\delta-\rho+\sigma)} s_{12}^2 \right) c_{23} s_{23} s_{13} \right] \frac{s_{12}}{c_{12}} \right. \\ & - \zeta_{12} c_{(\rho-\sigma)} \left[s_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(s_{(\delta+\rho-\sigma)} c_{12}^2 + s_{(\delta-\rho+\sigma)} s_{12}^2 \right) c_{23} s_{23} s_{13} \right] \frac{s_{12}}{c_{12}} \\ & + \left[\frac{s_\rho}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) + \zeta_{13} c_\rho \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \right] \frac{c_{23} (c_{12}^2 c_{13}^2 - s_{13}^2)}{c_{12} s_{13}} \\ & \left. - \left[\frac{s_\sigma}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) + \zeta_{23} c_\sigma \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right] \frac{c_{23} c_{13}^2 s_{12}}{s_{13}} \right\}, \\ \frac{d\sigma}{dt} = & \frac{y_\tau^2}{16\pi^2} \left\{ \frac{s_{(\rho-\sigma)}}{\zeta_{12}} \left[c_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(c_{(\delta+\rho-\sigma)} c_{12}^2 - c_{(\delta-\rho+\sigma)} s_{12}^2 \right) c_{23} s_{23} s_{13} \right] \frac{c_{12}}{s_{12}} \right. \\ & - \zeta_{12} c_{(\rho-\sigma)} \left[s_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) - \left(s_{(\delta+\rho-\sigma)} c_{12}^2 + s_{(\delta-\rho+\sigma)} s_{12}^2 \right) c_{23} s_{23} s_{13} \right] \frac{c_{12}}{s_{12}} \\ & - \left[\frac{s_\sigma}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) + \zeta_{23} c_\sigma \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right] \frac{c_{23} (c_{13}^2 s_{12}^2 - s_{13}^2)}{s_{12} s_{13}} \\ & \left. + \left[\frac{s_\rho}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) + \zeta_{13} c_\rho \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \right] \frac{c_{12} c_{23} c_{13}^2}{s_{13}} \right\}. \quad (7) \end{aligned}$$

It is worth mentioning that the analytical results in Eqs. (6) and (7) can also be achieved from Eqs. (14)–(19) of Ref. [15] by setting $C_\kappa^l = 1$ and $y_\nu = 0$ over there. One can see

that the running effects of three flavor mixing angles and three CP-violating phases are all proportional to the factor $y_\tau^2/(16\pi^2) = m_\tau^2(1 + \tan^2 \beta)/(16\pi^2 v^2) \approx 6.6 \times 10^{-7}(1 + \tan^2 \beta)$ in the MSSM. Hence an appreciable value of $\tan \beta$ is required, in order to get appreciable radiative corrections to relevant physical parameters.

Next let us consider three rephasing-invariant quantities of V . The first one is the Jarlskog parameter \mathcal{J} [18], defined through

$$\text{Im} \left(V_{\alpha i} V_{\beta j} V_{\alpha j}^* V_{\beta i}^* \right) = \mathcal{J} \sum_{\gamma, k} \left(\epsilon_{\alpha \beta \gamma} \epsilon_{ijk} \right), \quad (8)$$

where the Greek and Latin subscripts run over (e, μ, τ) and $(1, 2, 3)$, respectively. Taking account of the parametrization of V in Eq. (1), we have $\mathcal{J} = s_{12} c_{12} s_{23} c_{23} s_{13} c_{13}^2 s_\delta$. The off-diagonal asymmetries of V [19],

$$\begin{aligned} \mathcal{A}_L &\equiv |V_{e2}|^2 - |V_{\mu 1}|^2 = |V_{\mu 3}|^2 - |V_{\tau 2}|^2 = |V_{\tau 1}|^2 - |V_{e3}|^2, \\ \mathcal{A}_R &\equiv |V_{e2}|^2 - |V_{\mu 3}|^2 = |V_{\mu 1}|^2 - |V_{\tau 2}|^2 = |V_{\tau 3}|^2 - |V_{e1}|^2, \end{aligned} \quad (9)$$

are also rephasing-invariant. We obtain $\mathcal{A}_L = s_{12}^2 (c_{13}^2 - c_{23}^2) - c_{12}^2 s_{23}^2 s_{13}^2 - 2s_{12} c_{12} s_{23} c_{23} s_{13} c_\delta$ and $\mathcal{A}_R = c_{13}^2 (s_{12}^2 - s_{23}^2)$. The RGEs of \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R can then be derived from Eqs. (6) and (7). The result for \mathcal{J} is

$$\begin{aligned} \frac{d\mathcal{J}}{dt} = \frac{y_\tau^2}{16\pi^2} \Bigg\{ & -\frac{c_{23} c_{13}^2 s_{23} s_{13}}{\zeta_{12}} \left[c_\delta s_{(\rho-\sigma)} + c_{(\rho-\sigma)} s_\delta (c_{12}^2 - s_{12}^2) \right] \left[c_{(\delta+\rho-\sigma)} c_{12}^2 c_{23} s_{23} s_{13} \right. \\ & \left. - c_{(\delta-\rho+\sigma)} c_{23} s_{12}^2 s_{23} s_{13} - c_{(\rho-\sigma)} c_{12} s_{12} (s_{23}^2 - c_{23}^2 s_{13}^2) \right] \\ & + \zeta_{12} c_{23} c_{13}^2 s_{23} s_{13} \left[c_\delta c_{(\rho-\sigma)} - s_\delta s_{(\rho-\sigma)} (c_{12}^2 - s_{12}^2) \right] \left[s_{(\delta+\rho-\sigma)} c_{12}^2 c_{23} s_{23} s_{13} \right. \\ & \left. + s_{(\delta-\rho+\sigma)} c_{23} s_{12}^2 s_{23} s_{13} - s_{(\rho-\sigma)} c_{12} s_{12} (s_{23}^2 - c_{23}^2 s_{13}^2) \right] \\ & + \frac{c_{23} c_{13}^2 s_{12}}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) \left[s_\rho c_{12} s_{12} s_{13} (c_{23}^2 - s_{23}^2) \right. \\ & \left. + s_{(\delta-\rho)} c_{23} s_{12}^2 s_{23} s_{13}^2 + \left[c_\delta s_\rho - c_\rho s_\delta (c_{13}^2 - s_{13}^2) \right] c_{12}^2 c_{23} s_{23} \right] \\ & + \zeta_{13} c_{23} c_{13}^2 s_{12} \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \left[c_\rho c_{12} s_{12} s_{13} (c_{23}^2 - s_{23}^2) \right. \\ & \left. - c_{(\delta-\rho)} c_{23} s_{12}^2 s_{23} s_{13}^2 + \left[c_\delta c_\rho + s_\delta s_\rho (c_{13}^2 - s_{13}^2) \right] c_{12}^2 c_{23} s_{23} \right] \\ & - \frac{c_{12} c_{23} c_{13}^2}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) \left[\left(c_\delta s_\sigma - c_\sigma s_\delta c_{13}^2 \right) c_{23} s_{12}^2 s_{23} \right. \\ & \left. - s_\sigma c_{12} s_{12} s_{13} (c_{23}^2 - s_{23}^2) + \left(s_{(\delta-\sigma)} c_{12}^2 + c_\sigma s_\delta s_{12}^2 \right) c_{23} s_{23} s_{13}^2 \right] \\ & + \zeta_{23} c_{12} c_{23} c_{13}^2 \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \left[- \left(c_\delta c_\sigma + s_\delta s_\sigma c_{13}^2 \right) c_{23} s_{12}^2 s_{23} \right. \\ & \left. + c_\sigma c_{12} s_{12} s_{13} (c_{23}^2 - s_{23}^2) + \left(c_{(\delta-\sigma)} c_{12}^2 + s_\delta s_\sigma s_{12}^2 \right) c_{23} s_{23} s_{13}^2 \right] \Bigg\}. \end{aligned} \quad (10)$$

Different from $d\delta/dt$, $d\mathcal{J}/dt$ does not suffer from any divergence in the $\theta_{13} \rightarrow 0$ limit. This feature proves that \mathcal{J} itself is a well-defined rephasing-invariant quantity, while δ is parametrization-dependent and cannot be well defined when θ_{13} vanishes. The RGEs of \mathcal{A}_L and \mathcal{A}_R read as follows:

$$\begin{aligned}
\frac{d\mathcal{A}_L}{dt} = & \frac{y_\tau^2}{16\pi^2} \left\{ \frac{2}{\zeta_{12}} \left[c_{(\delta+\rho-\sigma)} c_{12}^2 c_{23} s_{23} s_{13} - c_{(\delta-\rho+\sigma)} c_{23} s_{12}^2 s_{23} s_{13} - c_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) \right] \right. \\
& \left[-s_\delta s_{(\rho-\sigma)} c_{23} s_{23} s_{13} + c_\delta c_{(\rho-\sigma)} c_{23} s_{23} s_{13} \left(c_{12}^2 - s_{12}^2 \right) - c_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) \right] \\
& + 2\zeta_{12} \left[s_{(\delta+\rho-\sigma)} c_{12}^2 c_{23} s_{23} s_{13} + s_{(\delta-\rho+\sigma)} c_{23} s_{12}^2 s_{23} s_{13} - s_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) \right] \\
& \left[c_{(\rho-\sigma)} s_\delta c_{23} s_{23} s_{13} + c_\delta s_{(\rho-\sigma)} c_{23} \left(c_{12}^2 - s_{12}^2 \right) s_{23} s_{13} - s_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) \right] \\
& + \frac{2c_{23}c_{13}^2s_{23}}{\zeta_{13}} \left(c_{(\delta-\rho)} s_{12} s_{23} - c_\rho c_{12} c_{23} s_{13} \right) \left(c_{(\delta-\rho)} c_{23} s_{12} + c_\rho c_{12} s_{23} s_{13} \right) \\
& + 2\zeta_{13} c_{23} c_{13}^2 s_{23} \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \left(s_{(\delta-\rho)} c_{23} s_{12} - s_\rho c_{12} s_{23} s_{13} \right) \\
& - \frac{2c_\sigma c_{23} c_{13}^2 s_{12} s_{13}}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right) \\
& \left. + 2\zeta_{23} s_\sigma c_{23} c_{13}^2 s_{12} s_{13} \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right) \right\} , \\
\frac{d\mathcal{A}_R}{dt} = & \frac{y_\tau^2 c_{13}^2}{16\pi^2} \left\{ \frac{2c_{(\rho-\sigma)} c_{12} s_{12}}{\zeta_{12}} \left[-c_{(\delta+\rho-\sigma)} c_{12}^2 c_{23} s_{23} s_{13} + c_{(\delta-\rho+\sigma)} c_{23} s_{12}^2 s_{23} s_{13} \right. \right. \\
& \left. + c_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) \right] - 2\zeta_{12} s_{(\rho-\sigma)} c_{12} s_{12} \left[s_{(\delta+\rho-\sigma)} c_{12}^2 c_{23} s_{23} s_{13} \right. \\
& \left. + s_{(\delta-\rho+\sigma)} c_{23} s_{12}^2 s_{23} s_{13} - s_{(\rho-\sigma)} c_{12} s_{12} \left(s_{23}^2 - c_{23}^2 s_{13}^2 \right) \right] \\
& + \frac{2c_{23}s_{23}}{\zeta_{13}} \left(-c_{(\delta-\rho)} s_{12} s_{23} + c_\rho c_{12} c_{23} s_{13} \right) \left(c_{(\delta-\rho)} c_{23} s_{12} + c_\rho c_{12} s_{23} s_{13} \right) \\
& + 2\zeta_{13} c_{23} s_{23} \left(s_{(\delta-\rho)} s_{12} s_{23} + s_\rho c_{12} c_{23} s_{13} \right) \left(-s_{(\delta-\rho)} c_{23} s_{12} + s_\rho c_{12} s_{23} s_{13} \right) \\
& \left. - \frac{2c_{23}^2}{\zeta_{23}} \left(c_{(\delta-\sigma)} c_{12} s_{23} + c_\sigma c_{23} s_{12} s_{13} \right)^2 - 2\zeta_{23} c_{23}^2 \left(s_{(\delta-\sigma)} c_{12} s_{23} - s_\sigma c_{23} s_{12} s_{13} \right)^2 \right\} . \quad (11)
\end{aligned}$$

Eqs. (10) and (11) clearly show that the RGE evolution of \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R depends on the Majorana phases ρ and σ . Hence the radiative corrections to these rephasing-invariant parameters would in general be quite different, if neutrinos were Dirac particles instead of Majorana particles ³.

III. NUMERICAL EXAMPLES AND DISCUSSION

We proceed to illustrate the radiative generation of three CP-violating phases by taking a few typical numerical examples. The eigenvalues of Y_l at Λ_{SS} are chosen in such a way that they can correctly run to their low-energy values [21] ⁴. We assume the masses of three

³See, e.g., Ref. [20] for a recent analysis of the RGE evolution of Dirac neutrino masses and lepton flavor mixing parameters.

⁴A similar treatment is required for the gauge couplings, the quark Yukawa coupling eigenvalues and the quark flavor mixing parameters in the full set of RGEs [11–15,22].

light neutrinos to be nearly degenerate and $m_1 \sim 0.2$ eV, so as to make the RGE running effects of relevant physical quantities significant enough. The initial values of κ_1 , κ_2 and κ_3 can be adjusted via

$$\begin{aligned}\kappa_1 &= \frac{m_1}{v^2 \sin^2 \beta}, \\ \kappa_2 &= \frac{\sqrt{m_1^2 + \Delta m_{21}^2}}{v^2 \sin^2 \beta}, \\ \kappa_3 &= \frac{\sqrt{m_1^2 + \Delta m_{31}^2}}{v^2 \sin^2 \beta},\end{aligned}\tag{12}$$

together with a typical input $\tan \beta = 10$, such that the resultant neutrino mass-squared differences $\Delta m_{21}^2 \equiv m_2^2 - m_1^2$ and $\Delta m_{31}^2 \equiv m_3^2 - m_1^2$ at Λ_{EW} are consistent with the solar and atmospheric neutrino oscillation data [1–5]. We follow a similar strategy to choose the initial values of three mixing angles θ_{12} , θ_{23} and θ_{13} , in order to reproduce their low-energy values determined or constrained from a global analysis of current experimental data [8]. In view of the upper bound $\theta_{13} < 10^\circ$, we shall typically take $\theta_{13} = 1^\circ$, 3° and 5° in our numerical calculations. We allow one of three CP-violating phases (δ , ρ , σ) to vanish at Λ_{SS} and examine whether it can run to 90° at Λ_{EW} by choosing the initial values of the other two phase parameters properly.

A. Radiative Generation of $\delta = 90^\circ$

First of all, let us look at the one-loop RGE evolution of δ in the $m_1 < m_2 < m_3$ case (i.e., $\Delta m_{31}^2 > 0$). The input and output values of relevant physical parameters are listed in Table I. One can see that $\delta = 90^\circ$ at Λ_{EW} can be radiatively generated from $\delta = 0^\circ$ at Λ_{SS} , if $\theta_{13} = 1^\circ$, $\rho = 4.0^\circ$ and $\sigma = -57.5^\circ$ are input. Changing the initial value of θ_{13} to 3° or 5° but fixing the input values of the other quantities, we find that only $\delta = 41.8^\circ$ or $\delta = 35.8^\circ$ can be obtained at Λ_{EW} . While the results of m_1 , Δm_{31}^2 , θ_{12} and θ_{23} at Λ_{EW} are rather stable against the change of θ_{13} from 1° to 5° at Λ_{SS} , the result of Δm_{21}^2 becomes smaller and less favored.

The running behaviors of δ , ρ and σ are explicitly shown in FIG. 1, in which those of \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R are also illustrated. Some comments are in order.

- The one-loop RGE running behaviors of three CP-violating phases are quite similar in the chosen parameter space. It is easy to understand this feature from Eq. (7): given the conditions that $\kappa_1 \approx \kappa_2 \approx \kappa_3$ holds and θ_{13} is small, Eq. (7) can be simplified to

$$\begin{aligned}\frac{d\delta}{dt} &\approx \frac{y_\tau^2}{16\pi^2} \left[\frac{c_{(\rho-\sigma)} s_{(\rho-\sigma)}}{\zeta_{12}} s_{23}^2 + \left(\frac{c_{(\delta-\rho)} s_\rho}{\zeta_{13}} - \frac{c_{(\delta-\sigma)} s_\sigma}{\zeta_{23}} \right) \frac{c_{12} s_{12} c_{23} s_{23}}{s_{13}} \right], \\ \frac{d\rho}{dt} &\approx \frac{y_\tau^2}{16\pi^2} \left[\frac{c_{(\rho-\sigma)} s_{(\rho-\sigma)}}{\zeta_{12}} s_{12}^2 s_{23}^2 + \left(\frac{c_{(\delta-\rho)} s_\rho}{\zeta_{13}} - \frac{c_{(\delta-\sigma)} s_\sigma}{\zeta_{23}} \right) \frac{c_{12} s_{12} c_{23} s_{23}}{s_{13}} \right], \\ \frac{d\sigma}{dt} &\approx \frac{y_\tau^2}{16\pi^2} \left[\frac{c_{(\rho-\sigma)} s_{(\rho-\sigma)}}{\zeta_{12}} c_{12}^2 s_{23}^2 + \left(\frac{c_{(\delta-\rho)} s_\rho}{\zeta_{13}} - \frac{c_{(\delta-\sigma)} s_\sigma}{\zeta_{23}} \right) \frac{c_{12} s_{12} c_{23} s_{23}}{s_{13}} \right].\end{aligned}\tag{13}$$

Thus the trend of ρ or σ in the RGE evolution is very similar to that of δ .

- The Jarlskog parameter is sensitive to both δ and θ_{13} . Given the initial condition $\theta_{13} = 1^\circ$, $\delta = 90^\circ$ at Λ_{EW} can be achieved from $\delta = 0^\circ$ at Λ_{SS} . It turns out that $\mathcal{J} \approx 0.26\%$ can be radiatively generated in this specific case. When $\theta_{13} = 5^\circ$ is input at Λ_{SS} , one may arrive at $\mathcal{J} \approx 1\%$ at Λ_{EW} , a value which might be detectable in the future long-baseline neutrino oscillation experiments.
- The off-diagonal asymmetries \mathcal{A}_L and \mathcal{A}_R are both non-vanishing in our numerical example. While \mathcal{A}_L is sensitive to the input of θ_{13} , \mathcal{A}_R is not. The reason for the latter feature is simply that $\mathcal{A}_R = c_{13}^2(s_{12}^2 - s_{23}^2)$ holds.

It is worth mentioning that the unitarity of V requires that its six unitarity triangles in the complex plane have the same area, amounting to $\mathcal{J}/2$ [22]. Thus the radiative generation of δ or \mathcal{J} geometrically implies the radiative generation of every leptonic unitarity triangle; i.e., three overlapped lines (sides) at Λ_{SS} can evolve into a triangle at Λ_{EW} .

Now let us examine whether the radiative generation of $\delta = 90^\circ$ at Λ_{EW} can be achieved from other initial values of ρ and σ , when $\theta_{13} = 1^\circ$ holds and m_1 , Δm_{21}^2 , Δm_{31}^2 , θ_{12} and θ_{23} take the same input values as before at Λ_{SS} (see Table I). We find out two new numerical examples with $(\rho, \sigma) = (0^\circ, -59.9^\circ)$ and $(10^\circ, -57^\circ)$, respectively, as shown in FIG. 2. Note that both δ and ρ (or σ) can be radiatively generated from $\sigma \neq 0^\circ$ (or $\rho \neq 0^\circ$) at the seesaw scale⁵. Our results hint at the existence of strong parameter degeneracy in obtaining $\delta = 90^\circ$ at Λ_{EW} from $\delta = 0^\circ$ at Λ_{SS} . To resolve this problem is certainly a big challenge in model building, unless two Majorana CP-violating phases could separately be measured at low energies.

Finally we demonstrate that it is also possible to radiatively generate $\delta = 90^\circ$ at Λ_{EW} from $\delta = 0^\circ$ at Λ_{SS} in the $\Delta m_{31}^2 < 0$ case. Our numerical results are shown in FIG. 3, where the initial values of relevant parameters are listed in Table II. One can see that the one-loop RGE running behaviors of three CP-violating phases in FIG. 3 are very similar to those in FIG. 1, although the initial conditions in these two cases are quite different. The reason for this similarity has actually been reflected by Eq. (13), where the sign of Δm_{31}^2 does not play a role in the leading-order approximation.

B. Radiative Generation of $\rho = 90^\circ$

We continue to take a look at the one-loop RGE evolution of ρ in the $m_1 < m_2 < m_3$ case (i.e., $\Delta m_{31}^2 > 0$). The input and output values of relevant physical parameters are listed in Table III. One can see that there is no difficulty to radiatively generate $\rho = 90^\circ$ at Λ_{EW} from $\rho = 0^\circ$ at Λ_{SS} , provided $\theta_{13} = 1^\circ$, $\delta = 0^\circ$ and $\sigma = -67.7^\circ$ are input. Allowing the

⁵The possibility to simultaneously generate ρ and σ from $\delta \neq 0^\circ$ at Λ_{SS} via the RGE evolution is in general expected to be strongly suppressed, because the leading terms of $d\rho/dt$ and $d\sigma/dt$ in Eq. (7) vanish for $\rho = \sigma = 0^\circ$ at Λ_{SS} . Only in the $\theta_{13} \rightarrow 0$ limit, the running effects of three CP-violating phases could become significant [15].

initial value of θ_{13} to change to 3° or 5° but fixing the input values of the other quantities, we find that only $\rho = 17.6^\circ$ or $\rho = 12.1^\circ$ can be obtained at Λ_{EW} . Again the results of m_1 , Δm_{31}^2 , θ_{12} and θ_{23} at Λ_{EW} are quite stable against the change of θ_{13} from 1° to 5° at Λ_{SS} , but the result of Δm_{21}^2 becomes smaller.

The RGE running behaviors of three CP-violating phases (δ, ρ, σ) and three rephasing-invariant parameters $(\mathcal{J}, \mathcal{A}_L, \mathcal{A}_R)$ are explicitly shown in FIG. 4. Comparing this figure with FIG. 1, one may observe much similarity between them. There are two reasons for this similarity: first, the initial conditions in these two cases are not very different; second, the evolution of δ , ρ and σ is dominated by Eq. (13) in both cases.

C. Radiative Generation of $\sigma = 90^\circ$

Finally let us look at the RGE evolution of σ in the $m_1 < m_2 < m_3$ case (i.e., $\Delta m_{31}^2 > 0$). The input and output values of relevant physical parameters are listed in Table IV. One can see that there is no difficulty to radiatively generate $\sigma = 90^\circ$ at Λ_{EW} from $\sigma = 0^\circ$ at Λ_{SS} , if $\theta_{13} = 1^\circ$, $\delta = 119.7^\circ$ and $\rho = 60.8^\circ$ are input. Allowing the initial value of θ_{13} to change to 3° or 5° but fixing the input values of the other quantities, we find that only $\sigma = 34.4^\circ$ or $\sigma = 29.8^\circ$ can be obtained at Λ_{EW} . The results of m_1 , Δm_{31}^2 , θ_{12} and θ_{23} at Λ_{EW} are very stable against the change of θ_{13} from 1° to 5° at Λ_{SS} , but the result of Δm_{21}^2 becomes larger and less favored.

The RGE running behaviors of δ , ρ and σ are shown in FIG. 5, in which those of \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R are also illustrated. Again the evolution of three CP-violating phases is dominated by Eq. (13). Because δ evolves from the second quadrant to the third one during the RGE running, a flip of the sign of \mathcal{J} appears around $\delta = 180^\circ$. The signs of \mathcal{A}_L and \mathcal{A}_R keep unchanged from Λ_{SS} to Λ_{EW} , implying that the geometrical structure of V does not change in a significant way.

In all the cases discussed above, $\langle m \rangle_{ee} \sim m_1$ holds as a consequence of the approximate mass degeneracy of three light neutrinos. Indeed, the smallness of θ_{13} allows us to obtain an approximate expression of $\langle m \rangle_{ee}$:

$$\langle m \rangle_{ee} \approx m_1 \sqrt{1 - \sin^2 2\theta_{12} \sin^2(\rho - \sigma)} \in [m_1 \cos 2\theta_{12}, m_1] . \quad (14)$$

Given $m_1 \approx 0.2$ eV and $\theta_{12} \approx 33^\circ$ at low-energy scales, $\langle m \rangle_{ee}$ turns out to lie in the range $0.08 \text{ eV} \leq \langle m \rangle_{ee} \leq 0.2 \text{ eV}$, a result consistent with the present experimental upper limit $\langle m \rangle_{ee} < 0.38 \text{ eV}$ [8].

IV. SUMMARY

Taking account of a very useful parametrization of the 3×3 MNS matrix V , we have derived the one-loop RGEs for its three mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, three CP-violating phases (δ, ρ, σ) and three rephasing-invariant quantities $(\mathcal{J}, \mathcal{A}_L, \mathcal{A}_R)$. Particular attention has been paid to the radiative generation of leptonic CP violation, because the Dirac phase δ and the Majorana phases ρ and σ are entangled with one another in the RGE evolution from the seesaw scale Λ_{SS} to the electroweak scale Λ_{EW} . We have shown that $\delta = 90^\circ$ at Λ_{EW} can

be radiatively generated from $\delta = 0^\circ$ at Λ_{SS} in the minimal supersymmetric standard model, provided the light neutrino masses are nearly degenerate and the mixing angle θ_{13} is of $\mathcal{O}(1^\circ)$ or smaller. As for ρ and σ , it is also possible to radiatively generate $\rho = 90^\circ$ or $\sigma = 90^\circ$ at Λ_{EW} from $\rho = 0^\circ$ or $\sigma = 0^\circ$ at Λ_{SS} . An interesting feature of our numerical analysis is that the one-loop RGE running behaviors of three CP-violating phases are quite similar in the chosen parameter space, no matter whether the sign of the neutrino mass-squared difference Δm_{31}^2 is positive or negative. As an important by-product, the geometrical structure of V (i.e., its off-diagonal asymmetries and the area of its unitarity triangles in the complex plane) against radiative corrections has also been discussed.

At this point, it is worthwhile to summarize some remarkable differences between our analytical and numerical results and those obtained in Refs. [12,13]:

(1) The phase convention of V used in Refs. [12,13] leads to the apparent dependence of $\langle m \rangle_{ee}$ (the effective mass of the neutrinoless double-beta decay) on δ , thus it is ill to refer to δ as the “Dirac” CP-violating phase. Such an ambiguity has been avoided in our phase convention of V , which forbids δ to enter the expression of $\langle m \rangle_{ee}$. Our Majorana phases (ρ and σ) are related to the ones defined by Casas *et al* [12] (ϕ and ϕ') or by Antusch *et al* [13] (φ_1 and φ_2) as follows: $\varphi_1 = \phi = 2(\delta - \rho)$ and $\varphi_2 = \phi' = 2(\delta - \sigma)$. On the other hand, our Dirac phase δ is indistinguishable from their phase parameter δ in the description of leptonic CP violation in neutrino oscillations.

(2) Our RGEs for three mixing angles and three CP-violating phases are apparently different from those given in Refs. [12,13], just because we have taken a different and more instructive phase convention for V . In particular, the leading terms of $d\rho/dt$ and $d\sigma/dt$ (i.e., the terms proportional to $1/\zeta_{12}$ and $1/s_{13}$) are very similar to those of $d\delta/dt$, as one can see from Eq. (7) or Eq. (13). This result implies that three CP-violating phases have similar RGE running behaviors in our phase convention. In contrast, the RGE running effect on φ_1 (or ϕ) and φ_2 (or ϕ') is much weaker than that on δ in Refs. [12,13].

(3) The one-loop RGEs for \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R given in Eqs. (10) and (11) are new results, although they can be derived from Eqs. (6) and (7) in a straightforward way. These three quantities are actually rephasing-invariant or parametrization-independent. Hence we can get some more generic feeling about radiative corrections to the leptonic CP-violating effect and the geometrical structure of V from the RGE evolution of \mathcal{J} , \mathcal{A}_L and \mathcal{A}_R . This kind of study was not done in Refs. [12,13] or elsewhere.

(4) Different from those previous works, we have concentrated on the novel possibilities to radiatively generate $\delta = 90^\circ$, $\rho = 90^\circ$ or $\sigma = 90^\circ$ at Λ_{EW} from $\delta = 0^\circ$, $\rho = 0^\circ$ or $\sigma = 0^\circ$ at Λ_{SS} in our numerical exercises. Because of $\mathcal{J} \propto \sin \delta$, $\delta \sim 90^\circ$ at low energies is a necessary condition to achieve sufficiently large CP- and T -violating effects in neutrino oscillations. On the other hand, $\rho \sim 90^\circ$ or $\sigma \sim 90^\circ$ at low energies may result in a kind of large cancellation in $\langle m \rangle_{ee}$, implying the possible suppression of this unique experimental observable to identify the Majorana nature of massive neutrinos.

It is also worth remarking that our analysis is essentially independent of the specific textures of lepton Yukawa coupling matrices, thus it can be applied to the concrete work of model building. Since the elegant thermal leptogenesis mechanism is usually expected to work at the seesaw scale, a study of its consequences at low-energy scales is available by means of the one-loop RGEs that we have obtained. In other words, the RGEs may serve as a useful bridge to establish a kind of connection between the phenomena of CP

violation at low- and high-energy scales. It would be extremely interesting, in our opinion, if the phase parameter governing the strength of CP violation in a long-baseline neutrino oscillation experiment could be radiatively generated from those CP-violating phases which control the matter-antimatter asymmetry of our universe at the seesaw scale.

One of us (Z.Z.X.) is grateful to S. Zhou for very helpful discussions. This work was supported in part by the National Nature Science Foundation of China.

REFERENCES

- [1] SNO Collaboration, Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011301 (2002).
- [2] For a review, see: C.K. Jung *et al.*, Ann. Rev. Nucl. Part. Sci. **51**, 451 (2001).
- [3] KamLAND Collaboration, K. Eguchi *et al.*, Phys. Rev. Lett. **90**, 021802 (2003).
- [4] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998); Palo Verde Collaboration, F. Boehm *et al.*, Phys. Rev. Lett. **84**, 3764 (2000).
- [5] K2K Collaboration, M.H. Ahn *et al.*, Phys. Rev. Lett. **90**, 041801 (2003).
- [6] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [7] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **517**, 363 (2001); Z.Z. Xing, Int. J. Mod. Phys. A **19**, 1 (2004).
- [8] M. Maltoni, T. Schwetz, M.A. Tórtola, and J.W.F. Valle, New J. Phys. **6**, 122 (2004); A. Strumia and F. Vissani, hep-ph/0503246.
- [9] P. Minkowski, Phys. Lett. B **67**, 421 (1977); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by F. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in *Quarks and Leptons*, edited by M. Lévy *et al.* (Plenum, New York, 1980), p. 707; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).
- [10] P.H. Chankowski and Z. Pluciennik, Phys. Lett. B **316**, 312 (1993); K.S. Babu, C.N. Leung, and J. Pantaleone, Phys. Lett. B **319**, 191 (1993); M. Tanimoto, Phys. Lett. B **360**, 41 (1995).
- [11] For a recent review with extensive references, see: P.H. Chankowski and S. Pokorski, Int. J. Mod. Phys. A **17**, 575 (2002).
- [12] J.A. Casas, J.R. Espinosa, A. Ibarra, and I. Navarro, Nucl. Phys. B **573**, 652 (2000).
- [13] S. Antusch, J. Kersten, and M. Lindner, M. Ratz, Nucl. Phys. B **674**, 401 (2003).
- [14] S. Antusch, J. Kersten, M. Lindner, M. Ratz, M.A. Schmidt, JHEP **0503**, 024 (2005).
- [15] J.W. Mei, Phys. Rev. D **71**, 073012 (2005). At this point it is worthwhile to correct two typing errors in Appendix B of this paper: (1) Eq. (B3) should be $\dot{\kappa}' + T_{\kappa}\kappa' + \kappa'T_{\kappa}^T = \alpha_{\kappa}\kappa' + \tilde{N}_{\kappa}\kappa' + \kappa'\tilde{N}_{\kappa}^T$; and (2) Eq. (B5) should be $k_i = (\alpha_{\kappa} + 2\text{Re}\tilde{N}_{\kappa,ii})k_i$.
- [16] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
- [17] H. Fritzsch and Z.Z. Xing, Phys. Lett. B **353**, 114 (1995); Nucl. Phys. B **556**, 49 (1999).
- [18] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985).
- [19] Z.Z. Xing, Phys. Rev. D **65**, 113010 (2002).
- [20] M. Lindner, M. Ratz, and M.A. Schmidt, hep-ph/0506280; C.W. Chiang, Phys. Rev. D **63**, 076009 (2001).
- [21] Particle Data Group, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
- [22] H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. **45**, 1 (2000); and references therein.

TABLES

TABLE I. Radiative generation of $\delta = 90^\circ$ at the electroweak scale Λ_{EW} from $\delta = 0^\circ$ at the seesaw scale Λ_{SS} in the MSSM with $\tan\beta = 10$ and $\Delta m_{31}^2 > 0$.

| Parameter | Input (Λ_{SS}) | Output (Λ_{EW}) | | |
|---|---------------------------------|----------------------------------|-------------------------|-------------------------|
| | | $\theta_{13} = 1^\circ$ | $\theta_{13} = 3^\circ$ | $\theta_{13} = 5^\circ$ |
| $m_1(\text{eV})$ | 0.241 | 0.20 | 0.20 | 0.20 |
| $\Delta m_{21}^2(10^{-5} \text{ eV}^2)$ | 20.4 | 7.79 | 7.17 | 6.56 |
| $\Delta m_{31}^2(10^{-3} \text{ eV}^2)$ | 3.32 | 2.20 | 2.20 | 2.20 |
| θ_{12} | 24.1° | 33.0° | 33.0° | 33.1° |
| θ_{23} | 43.9° | 45.1° | 45.0° | 45.0° |
| θ_{13} | $1^\circ/3^\circ/5^\circ$ | 0.65° | 2.46° | 4.52° |
| δ | 0° | 90.0° | 41.8° | 35.8° |
| ρ | 4.0° | 72.2° | 23.8° | 17.6° |
| σ | -57.5° | 26.3° | -22.0° | -28.1° |

TABLE II. Radiative generation of $\delta = 90^\circ$ at the electroweak scale Λ_{EW} from $\delta = 0^\circ$ at the seesaw scale Λ_{SS} in the MSSM with $\tan\beta = 10$ and $\Delta m_{31}^2 < 0$.

| Parameter | Input (Λ_{SS}) | Output (Λ_{EW}) | | |
|---|---------------------------------|----------------------------------|-------------------------|-------------------------|
| | | $\theta_{13} = 1^\circ$ | $\theta_{13} = 3^\circ$ | $\theta_{13} = 5^\circ$ |
| $m_1(\text{eV})$ | 0.241 | 0.20 | 0.20 | 0.20 |
| $\Delta m_{21}^2(10^{-5} \text{ eV}^2)$ | 20.8 | 7.96 | 7.37 | 6.79 |
| $\Delta m_{31}^2(10^{-3} \text{ eV}^2)$ | -3.08 | -2.20 | -2.21 | -2.21 |
| θ_{12} | 25.1° | 33.3° | 33.5° | 33.7° |
| θ_{23} | 46.7° | 45.2° | 45.2° | 45.2° |
| θ_{13} | $1^\circ/3^\circ/5^\circ$ | 0.68° | 2.38° | 4.34° |
| δ | 0° | 90.0° | 43.1° | 36.8° |
| ρ | -81.9° | -13.6° | -60.7° | -67.2° |
| σ | 34.3° | 117.7° | 70.6° | 64.1° |

TABLE III. Radiative generation of $\rho = 90^\circ$ at the electroweak scale Λ_{EW} from $\rho = 0^\circ$ at the seesaw scale Λ_{SS} in the MSSM with $\tan\beta = 10$ and $\Delta m_{31}^2 > 0$.

| Parameter | Input (Λ_{SS}) | Output (Λ_{EW}) | | |
|---|---------------------------------|----------------------------------|-------------------------|-------------------------|
| | | $\theta_{13} = 1^\circ$ | $\theta_{13} = 3^\circ$ | $\theta_{13} = 5^\circ$ |
| $m_1(\text{eV})$ | 0.241 | 0.20 | 0.20 | 0.20 |
| $\Delta m_{21}^2(10^{-5} \text{ eV}^2)$ | 20.4 | 8.54 | 7.90 | 7.27 |
| $\Delta m_{31}^2(10^{-3} \text{ eV}^2)$ | 3.32 | 2.21 | 2.20 | 2.20 |
| θ_{12} | 27.6° | 33.1° | 33.2° | 33.3° |
| θ_{23} | 43.9° | 44.8° | 44.8° | 44.8° |
| θ_{13} | $1^\circ/3^\circ/5^\circ$ | 0.43° | 2.17° | 4.24° |
| δ | 0° | 107.6° | 35.4° | 30.2° |
| ρ | 0° | 90.2° | 17.6° | 12.1° |
| σ | -67.7° | 34.1° | -38.3° | -44.6° |

TABLE IV. Radiative generation of $\sigma = 90^\circ$ at the electroweak scale Λ_{EW} from $\sigma = 0^\circ$ at the seesaw scale Λ_{SS} in the MSSM with $\tan\beta = 10$ and $\Delta m_{31}^2 > 0$.

| Parameter | Input (Λ_{SS}) | Output (Λ_{EW}) | | |
|---|---------------------------------|----------------------------------|-------------------------|-------------------------|
| | | $\theta_{13} = 1^\circ$ | $\theta_{13} = 3^\circ$ | $\theta_{13} = 5^\circ$ |
| $m_1(\text{eV})$ | 0.241 | 0.20 | 0.20 | 0.20 |
| $\Delta m_{21}^2(10^{-5} \text{ eV}^2)$ | 20.3 | 8.24 | 8.85 | 9.50 |
| $\Delta m_{31}^2(10^{-3} \text{ eV}^2)$ | 3.32 | 2.21 | 2.21 | 2.21 |
| θ_{12} | 24.1° | 33.2° | 34.2° | 35.2° |
| θ_{23} | 43.9° | 44.6° | 44.6° | 44.6° |
| θ_{13} | $1^\circ/3^\circ/5^\circ$ | 0.51° | 2.27° | 4.29° |
| δ | 119.7° | 216.0° | 161.0° | 157.1° |
| ρ | 60.8° | 135.3° | 78.8° | 73.6° |
| σ | 0° | 90.0° | 34.4° | 29.8° |

FIGURES

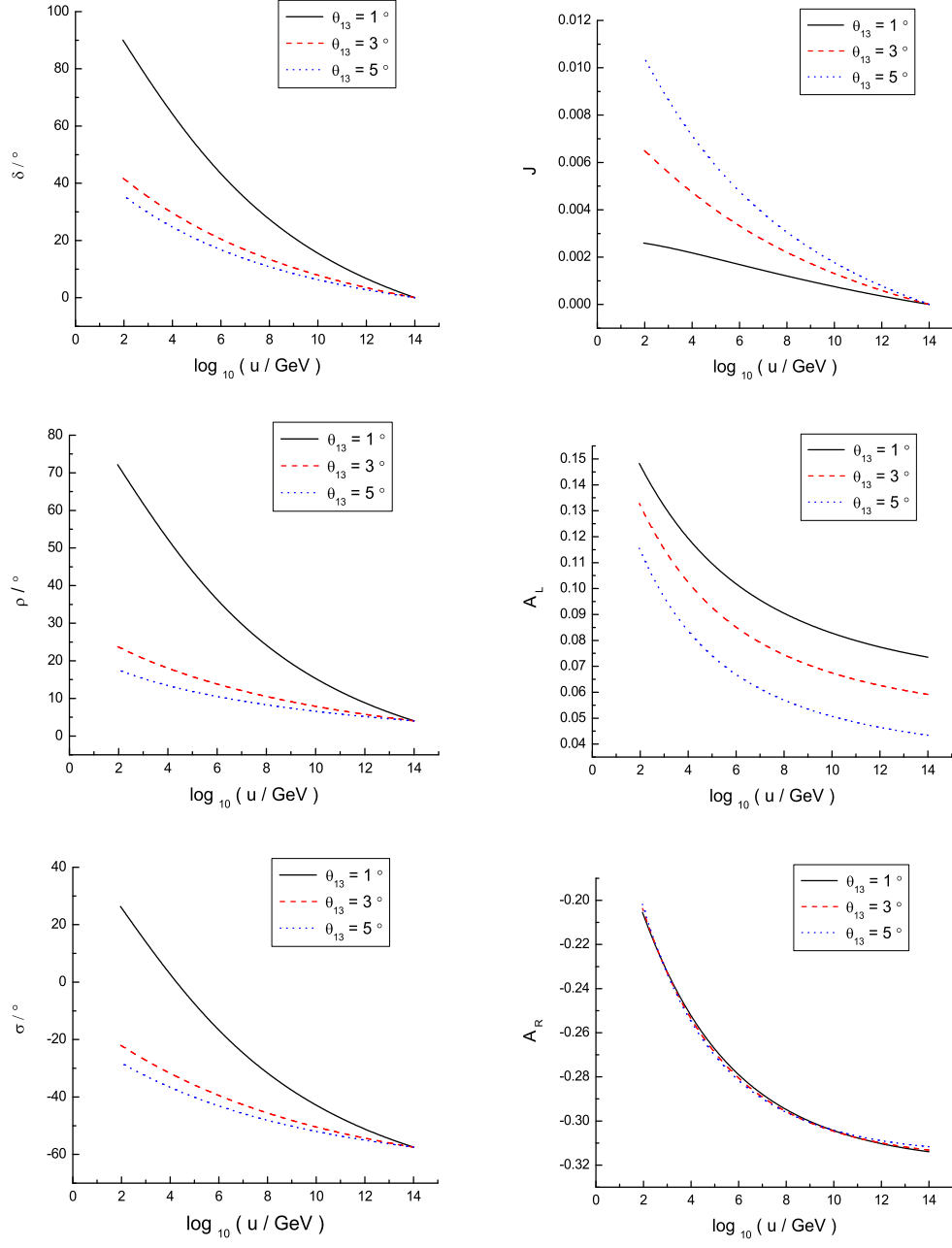


FIG. 1. The RGE running behaviors of three CP-violating phases and three rephasing-invariant quantities of V from Λ_{SS} to Λ_{EW} in the MSSM, where the input values of relevant parameters can be found from Table I.

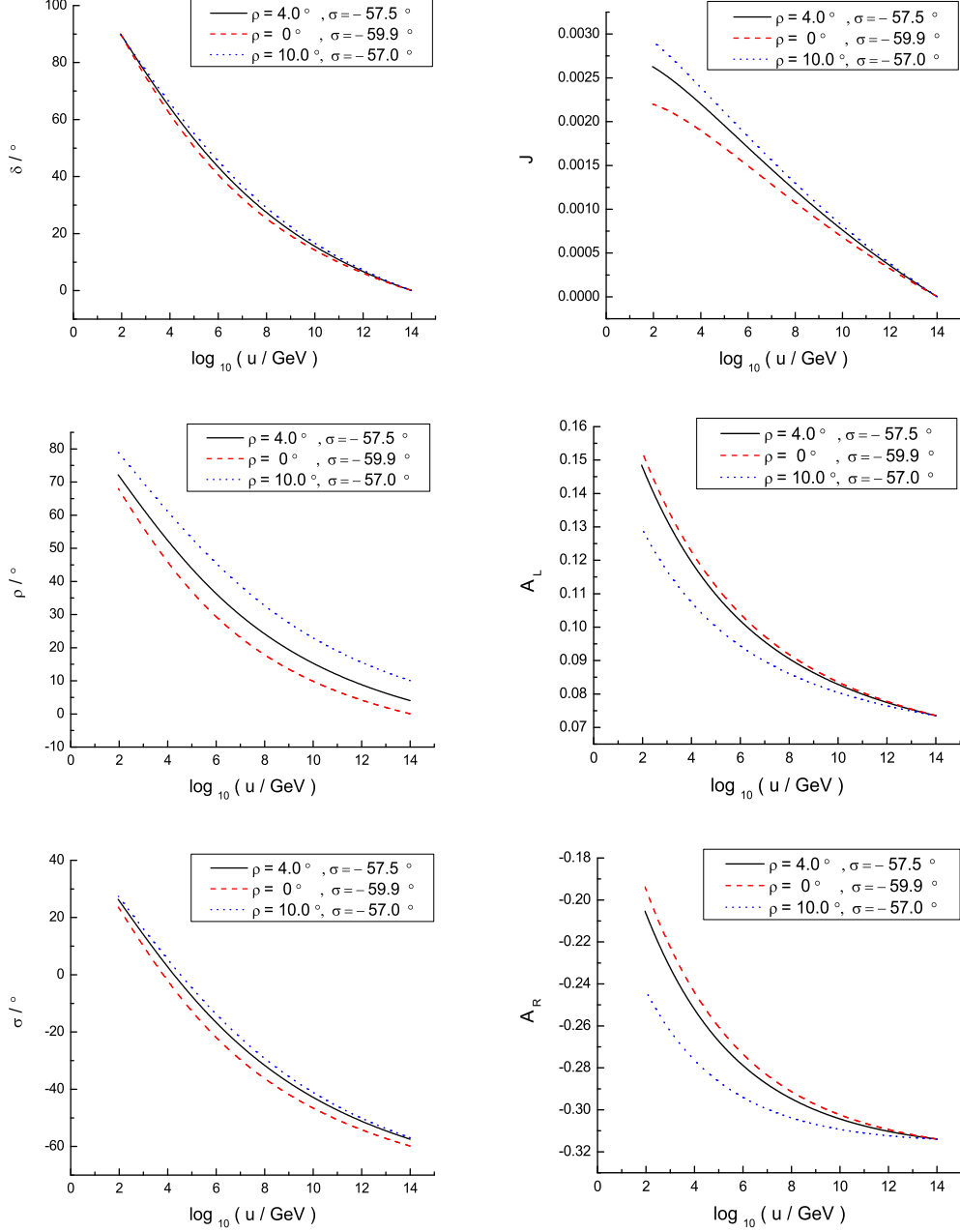


FIG. 2. The radiative generation of $\delta = 90^\circ$ at Λ_{EW} from $\theta_{13} = 1^\circ$ and different values of ρ and σ at Λ_{SS} . The input values of other parameters can be found from Table I.

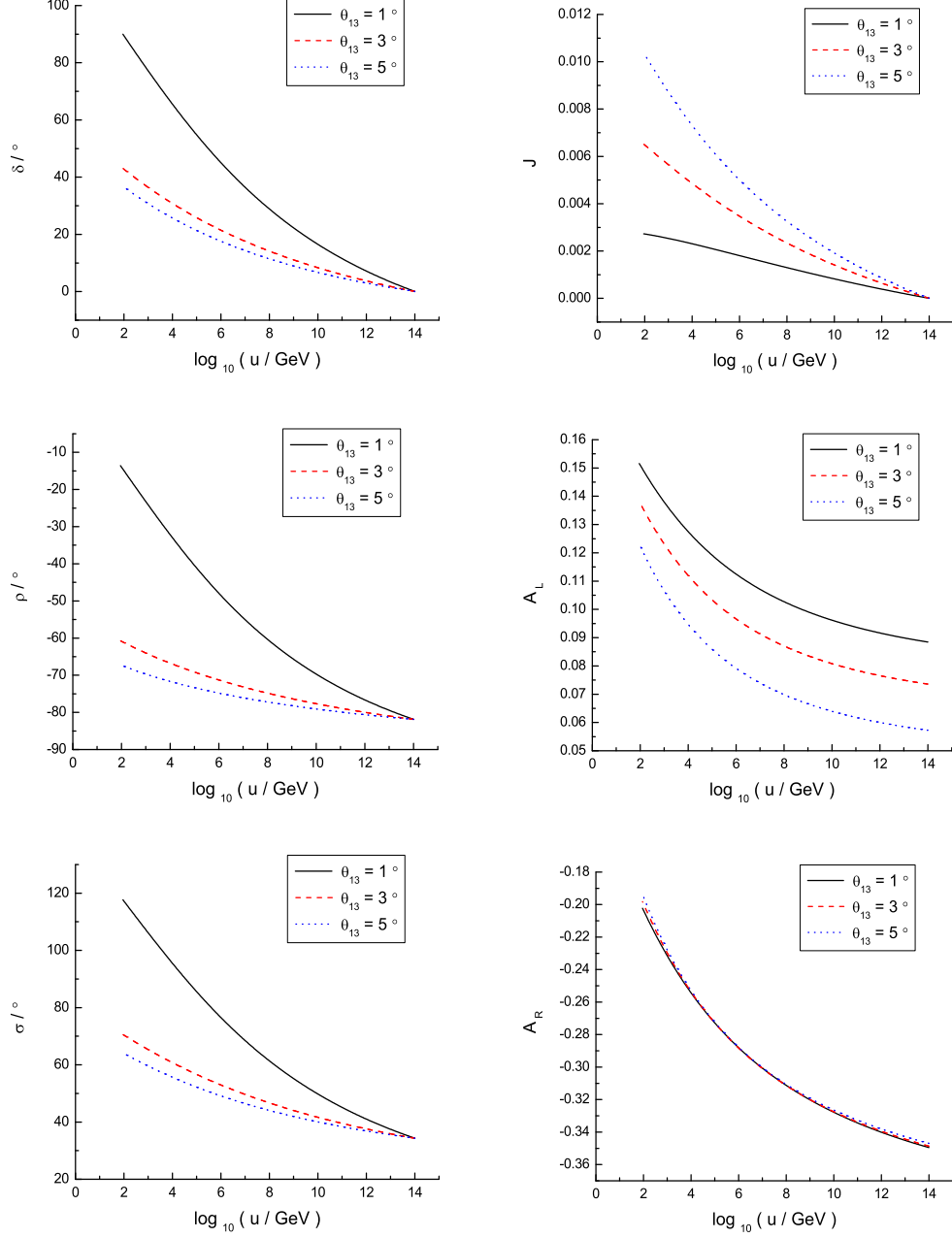


FIG. 3. The RGE running behaviors of three CP-violating phases and three rephasing-invariant quantities of V from Λ_{SS} to Λ_{EW} in the MSSM, where the input values of relevant parameters can be found from Table II.

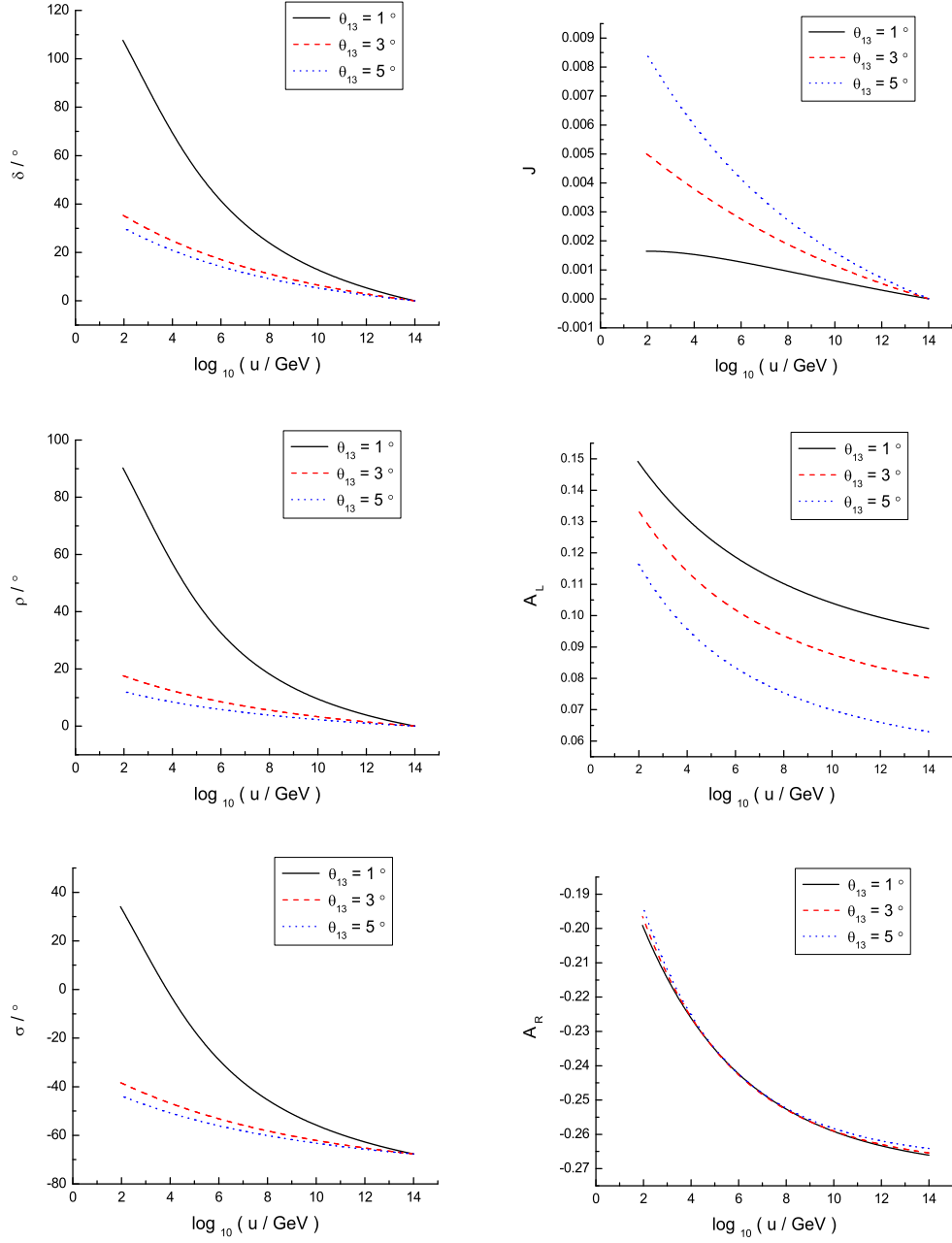


FIG. 4. The RGE running behaviors of three CP-violating phases and three rephasing-invariant quantities of V from Λ_{SS} to Λ_{EW} in the MSSM, where the input values of relevant parameters can be found from Table III.

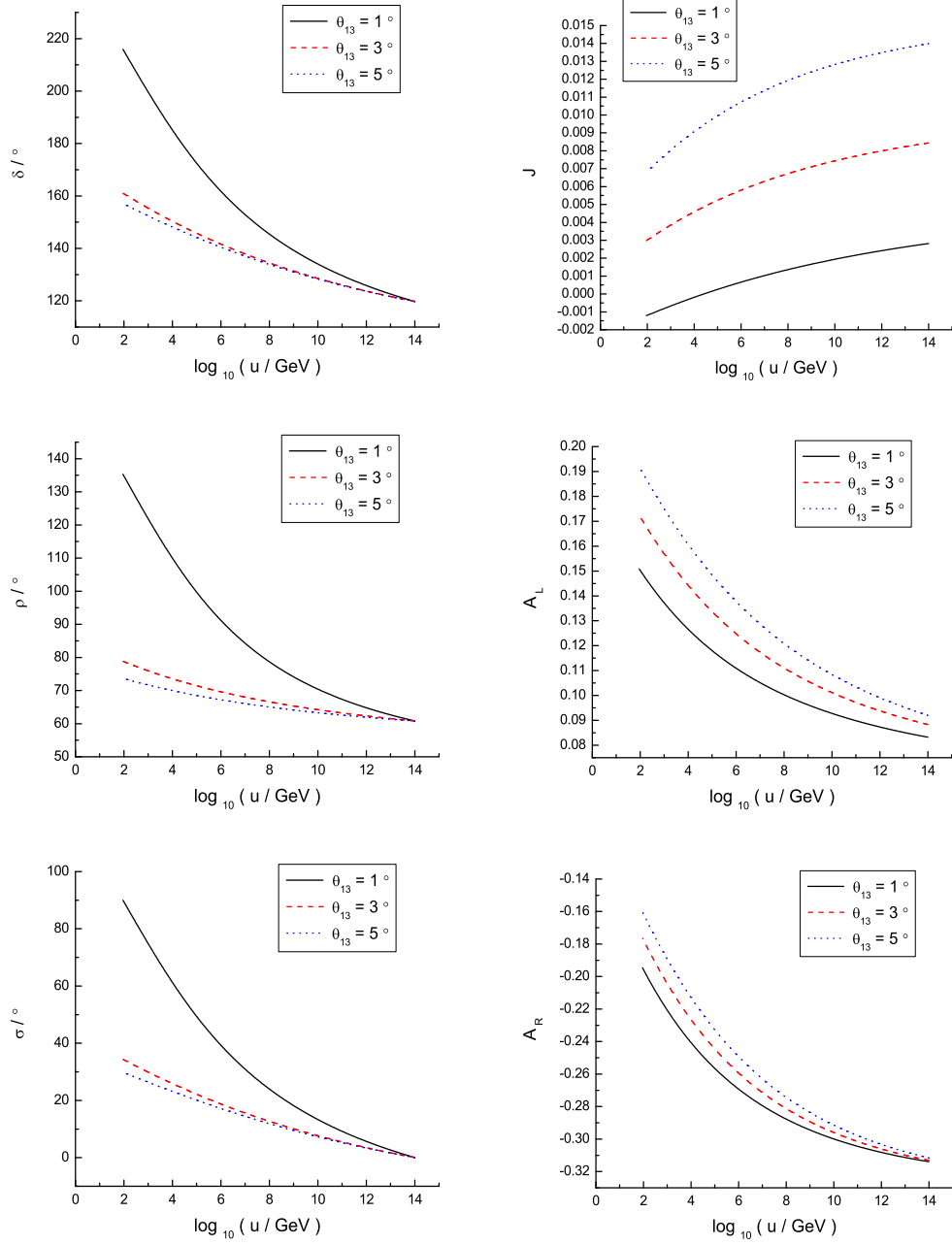


FIG. 5. The RGE running behaviors of three CP-violating phases and three rephasing-invariant quantities of V from Λ_{SS} to Λ_{EW} in the MSSM, where the input values of relevant parameters can be found from Table IV.